

# Particle Swarm Optimization for Simultaneous Optimization of Design and Machining Tolerances

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## 1. Introduction

Tolerance assignment in product design and process planning (machining) affects both the quality and the cost of the overall product cycle. It is a crucial issue to determine how much the tolerance should be relaxed during the assignment process, since a tight tolerance implies a high manufacturing cost and a loose tolerance results in low manufacturing cost. Hence, during tolerance assignment, a balance between a reduction in quality loss and a reduction in manufacturing cost must be considered. Traditionally, in the two stages (product design and process planning) tolerances (Ngoi & Teck, 1997) are often conducted separately. This is probably due to the fact that they deal with different type of tolerances. Product design is concerned with related component tolerances, whereas process designing focus on the process tolerance according to the process specification. However, this separated approach in tolerance design always suffers from several drawbacks. First of all, it is difficult to obtain optimal design tolerance because the designer can not determine the exact manufacturing cost without the specified manufacturing information. Therefore, the manufacturing engineer must frequently communicate with the designer to adjust the design tolerances and obtain the appropriate process planning. However, this task is time-consuming and painstaking. In addition, design tolerances are further distributed for machining tolerances. Nevertheless, the machining tolerances commonly can not occupy the design tolerances space. Thus the final tolerance distribution is suboptimal and accordingly, the actual cost will be inevitably higher than the desired cost. Moreover, due to the specified procedure, the manufacturing engineer is not informed the design details and does not have the overview of the whole product.

To overcome the above drawbacks, we need to develop a simultaneous tolerance design. Zhang (Zhang, 1996) presented the concept of simultaneous tolerance, proposed a general mathematical model for tolerance optimization in concurrent engineering context, and then introduced a new concept of interim tolerances that help determine appropriate manufacturing processes. Singh (Singh et al., 2003) utilized genetic algorithms and penalty function approach to solve the problem of simultaneous selection of design and manufacturing tolerances based on the minimization of the total manufacturing cost. Gao and Huang (Gao & Huang, 2003) utilized a nonlinear programming model for optimal

process tolerance simultaneously based on the objective of total manufacturing cost with different weighting factors. (Huang et al., 2006) proposed a robust optimum tolerance design method in a concurrent environment to balance the conflict design targets between manufacturing tolerances and product satisfaction. A nonlinear optimal model was also established to minimize the summation of manufacturing costs and product quality loss.

Doubtlessly, the tremendous achievement has been obtained in the simultaneous tolerance optimization in the concurrent engineering context. However, this problem is characterized by nonlinear objective, multiple independent variables, and tight constraints which will turn the search space into a noisy solution surface. Even worse, most of the real world problems become more and more complex with the higher requirement of accuracy and the critical function of product. Traditional operational research algorithms are successful in locating the optimal solution, but they are usually problem dependent and lack of generality. Some modern heuristic methods are relatively more robust and flexible to solve these complex problems, but they may risk being trapped to a local optimum and are usually slow in convergence and require heavy computational cost. In view of the above problems and the past successful applications of PSO in nonlinear optimization, maybe PSO is a potential remedy to these drawbacks.

PSO is a novel population based heuristic, which utilizes the swarm intelligence generated by the cooperation and competition between the particles in a swarm (Kennedy & Eberhart, 1995, Shi & Eberhart, 1998). Compared with evolutionary algorithms (genetic algorithm, evolutionary programming, evolutionary strategy, and genetic programming), PSO still maintains the population based global search strategy but adopts the velocity-displacement model with more efficient information flow and easier implementing procedures. It has been used successfully to address problems such as complex nonlinear function optimization (Shi & Eberhart, 1999), task assignment (Salman & Ahmad, 2002) and optimum design of PID controller (Gaing, 2004). (Noorul et al., 2006) utilized PSO to achieve the multiple objective of minimum quality loss function and manufacturing cost for the machining tolerance allocation of the over running clutch assembly. The presented method outperforms other methods such as GP and GA, but it considered only two dimensional tolerance allocation of clutch assembly consisting of three components. Besides, the constraints are too loose and can not satisfy the practical requirement. This paper attempts to solve more complex tolerance assignment problems by PSO with a sophisticated constraints handling strategy.

This paper is organized as follows. In section 2, the problem of simultaneous design was described. The basic PSO algorithm was reviewed and the new sophisticated constraints handling strategy corresponding to PSO was presented in Section 3. Section 4 gave an example and the evaluation of the proposed technique is carried out on the example. Some conclusions and further discussion are offered in Section 5.

## 2. Simultaneous design

As mentioned before, design processes are commonly divided into two main stages: product design and process design. Dimensional tolerance analysis is very important in both product and process design. In product design stage, the functional and assembly tolerances should be appropriately distributed among the constituent dimensions, this kind of tolerances are called design tolerances. In the meantime, each design tolerance for the single dimension should be subsequently refined to satisfy the requirement for process plans in

machining a part. Such tolerances for the specified machining operation are called manufacturing tolerance. However, the traditional process of design and machining tolerance allocations based on experiences can not guarantee optimum tolerance for minimum production cost. This work aimed at selecting the optimal tolerances sequences to achieve the minimum manufacturing cost considering the two types of tolerances simultaneously by a powerful global optimization tool. This problem is formulated as follows.

### 2.1 Objective Function

We take the manufacturing cost as the objective function. Generally, the processing of mechanical product is conducted in a series of process plans. Different process consumes different expense because different process is associated with different machining methods. Therefore, the cost of manufacture of the product is the summation of all operation cost. The machining operation can be modeled with many mathematical models for the cost-tolerance relationship. In this work, a modified form of the exponential cost (Singh et al., 2003) function will be adopted. The manufacturing cost of the machining tolerance is formulated as equation (1).

$$c_{ij}(\delta_{ij}) = a_0 e^{-a_1(\delta_{ij}-a_2)} + a_3 \quad i = 1, \dots, n \quad (1)$$

The total manufacturing cost of a product will be  $C$ , where:

$$C = \sum_{i=1}^n \sum_{j=1}^{m_i} c_{ij} \quad (2)$$

Where  $c_{ij}(\delta_{ij})$  and  $\delta_{ij}$  is the manufacturing cost and the tolerance of the  $j^{\text{th}}$  manufacturing operation associated with the  $i^{\text{th}}$  dimension respectively.  $n$  is the number of the dimensions and  $m_i$  is number of operations corresponding to dimension  $i$ . The constants  $a_0, a_1, a_2, a_3$  sever as control parameters.

### 2.2 Constraints

Apart from the constraint of economical manufacturing ranges (process limits), the above objective is subjected to both the design and manufacturing tolerances.

(1) The design tolerances are those on the principal design dimensions (usually assembly dimensions) that relate to the functionality of the components. The principal design usually in turn relies on the other related dimensions which form a dimension chain. This results in a set of constraints on the principal design tolerances that should be suit for the optimal solution of the tolerance assignment. The aim of these constraints is to guarantee that the synthesized tolerance in the dimension chain does not exceed the desired tolerance of the principal dimension. There are many approaches available to formulate the synthesized tolerance. They are different tradeoff between the tolerances and the manufacturing cost. Four commonly used approaches (Singh et al., 2003) were adopted in this work.

(2) Manufacturing tolerances constraints are equivalent to stock allowance constraints. Stock allowance is associated with the stock removal, the layer to be removed from the surface in the machining process. Due to the tolerances of the dimensions, the stock removal is also not

fixed. This gives rise to another kind of tolerances, manufacturing tolerances, which can be formulated as follows:

$$\delta_{ij} + \delta_{i(j-1)} \leq \Delta A_{ij} \quad (3)$$

where  $\delta_{ij}$  and  $\delta_{i(j-1)}$  are the machining tolerances of process  $j$  and  $j-1$  for part  $i$  respectively.  $\Delta A_{ij}$  is the difference between the nominal and the minimum machining allowances for machining process  $j$ .

### 3. Particle Swarm Optimization

#### 3.1 Background

The investigation and analysis on the biologic colony demonstrated that intelligence generated from complex activities such as cooperation and competition among individuals can provide efficient solutions for specific optimization problems (Kennedy et al., 2001). Inspired by the social behavior of animals such as fish schooling and bird flocking, Kennedy and Eberhart designed the Particle Swarm Optimization (PSO) in 1995 (Kennedy & Eberhart, 1995).

This method is a kind of evolutionary computing technology based on swarm intelligence. The basic idea of bird flocking can be depicted as follows: In a bird colony, each bird looks for its own food and in the meantime they cooperate with each other by sharing information among them. Therefore, each bird will explore next promising area by its own experience and experience from the others. Due to these attractive characteristics, i.e. memory and cooperation, PSO is widely applied in many research area and real-world engineering fields as a powerful optimization tool.

#### 3.2 Drawbacks of Traditional Constraints Handling Strategy

Although PSO has successfully solved many research problems, the applications are mainly focused on unconstrained optimization problems. Some researchers attempt to solve the constrained problem by optimizing constrained problems indirectly using the traditional penalty function strategy.

Penalty function is an effective auxiliary tool to deal with simple constrained problems and has been the most popular approach because of their simplicity and ease of implementation. Nevertheless, since the penalty function approach is generic and applicable to any type of constraint, their performance is not always satisfactory, especially when the problems become more difficult and the imposed constrained conditions become more complex, this method usually fails to generate the best solution, sometimes even cannot achieve a feasible one. The underlying limitation is that unfair competition exists in the population. Thus to deal with this problem, the dynamic and adaptive penalty coefficients should be introduced, which are highly dependent on the specific problem.

When combined with PSO, the above problem is more severe in that PSO has an inherent mechanism based on memory information. This mechanism can produce high efficiency and effectiveness, but also low the flexibility for constrained optimization simultaneously. That is, the penalty factors cannot be changed during the iteration. In fact, the most difficult aspect of the penalty function strategy is to find appropriate penalty parameters to guide the search towards the constrained optimum. It is desirable to design a new constraint handling

scheme suit for PSO to effectively solve numerous engineering problems and maintain high efficiency.

### 3.3 Constraints Handling Strategy for PSO

Taking account of the memory mechanism of PSO and penalty strategy, a new constraint-handling strategy is presented in Figure.1.

The core characteristics of the proposed strategy can be described as follows:

1. Corresponding to the memory mechanism of PSO, a special notation-Particle Feasible (PF) is introduced, which is used to record whether the current particle has ever satisfied all the constraint conditions. This notation preserves historical constrain status for each particle.
2. Each particle updates its individual best and neighborhood best according to the historical constraint information PF, the current constrain status (Current particle is Feasible, CF) and the objective function with the penalty term.
3. The algorithm selects the velocity updating strategy according to the historical information PF.
4. When updating the personal and neighborhood best, the algorithm adopts the static penalty strategy instead of the dynamic and the adaptive ones to guarantee the fairness. The detailed procedure for updating the personal and neighborhood best values based on the above constrain handling strategy is presented in Figure.1.

```

For Each Particle {
  If PF = true Then
    If  $f(x_i) \leq f(p_i)$  and CF= true Then
       $p_i = x_i$ 
    If  $f(p_i) \leq f(l_i)$  Then
       $p_i = l_i$ 
    End if
  End if
  Else if PF = false Then
    If CF = true Then
       $p_i = x_i$ 
      PF = true
      If  $f(p_i) \leq f(l_i)$  Then
         $p_i = l_i$ 
      End if
    Else if  $f(x_i) \leq f(p_i)$  Then
       $p_i = x_i$ 
    End if
  End if
}
    
```

Figure 1. The proposed constraint handling strategy for PSO

Special attention should be paid that the PSO algorithm based on the proposed constraint handling strategy does not have to guarantee the existence of feasible solutions in the initial population. With the randomized initial velocity, the PSO itself has the ability to explore the feasible space. In addition, the penalty function imposed on the violated particles also direct the search of PSO towards the feasible region. Therefore once feasible solutions emerge in the neighborhood population, the neighborhood best will be preserved in the subsequent iteration procedure. According to the velocity updating formula, each particle will obtain updating information from its neighborhood best particle, so the corresponding particle would return to the feasible solution space immediately.

#### 4. Design Example

To validate the effectiveness of the new proposed strategy and illustrate the application of the concurrent design, the cylinder-piston assembly (Singh et al., 2003) (shown in Figure.2) is described. In this example, the piston diameter is 50.8mm, the cylinder bore diameter is 50.856mm, and the clearance is  $0.056 \pm 0.025$  mm. The machining process plan is: (1) for the piston: rough turning, finish turning, rough grinding, and finally finish grinding. (2) for the cylinder bore: drilling, boring, semi-finish boring, and finally grinding. The ranges of the principal machining tolerances for the piston and cylinder bore were the same as in the (Singh et al., 2003).

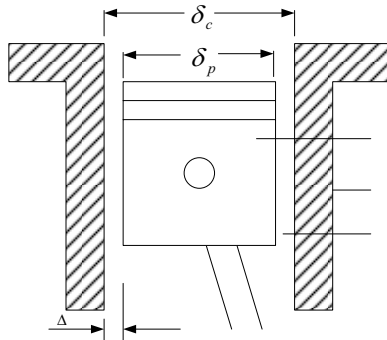


Figure 2. Cylinder-piston assembly

To formulate this problem, the objective and the constraints should be determined. In this problem, the principal tolerances are the design tolerances and the machining tolerances for the piston and the cylinder bore. So there are only two design tolerance parameters, for the piston diameter and cylinder bore diameter respectively. In the meantime, we have four machining tolerances for the piston diameter and four machining tolerances for the cylinder bore diameter. Therefore we have to consider totally 10 tolerances for the piston-cylinder bore assembly as follows. (1)The design tolerance parameters:  $\delta_{11d}$  for the piston and  $\delta_{21d}$  for the cylinder bore. Four stack-up conditions (Singh et al., 2003) (worst case, RSS, Spotts' modified method and estimated mean shift criteria) are employed to formulate the corresponding constraints. (2)The machining tolerance parameters are:  $\delta_{ij}$  where  $i=1,2$  and  $j=1,2,3,4$ . Here, the first subscript 1 refer to piston and 2 refer to the cylinder bore. The second subscript refers to the four machining processes. Usually, the process tolerance for

the final finishing operation is same as the design tolerance, i.e.  $\delta_{11d} = \delta_{14}$  and  $\delta_{12d} = \delta_{24}$ . Thus, there are actually 8 tolerance parameters to be considered. The machining tolerance constraints are formulated based on Equation 3. The manufacturing decision is the total machining cost and is determined by summing the machining cost-tolerance model as Equation 1 and Equation 2 subjecting to the constraints and ranges of the principal design and machining tolerances. The constant parameters are the same as in (Singh et al., 2003).

GA				PSO					
Piston	Cylinder	Cost	Time (s)	Piston	Cylinder	Min	Ave	Max	Time (s)
0.0162	0.0162	66.85	350	0.0163	0.0163	66.74	66.74	66.74	83
0.0037	0.0038			0.0037	0.0037				
0.0013	0.0012			0.0013	0.0013				
0.0005	0.0005			0.0005	0.0005				

(a) Based on the worst case criteria

GA				PSO					
Piston	Cylinder	Cost	Time (s)	Piston	Cylinder	Min	Ave	Max	Time (s)
0.0161	0.0161	65.92	330	0.0161	0.0162	66.82	66.82	66.82	80
0.0039	0.0038			0.0039	0.0038				
0.0011	0.0012			0.0011	0.0012				
0.0007	0.0006			0.0007	0.0006				

(b) Based on the worst RSS criteria

GA				PSO					
Piston	Cylinder	Cost	Time (s)	Piston	Cylinder	Min	Ave	Max	Time (s)
0.0160	0.0159	66.23	330	0.0162	0.0162	65.93	65.93	65.93	78
0.0038	0.0038			0.0038	0.0038				
0.0012	0.0012			0.0012	0.0012				
0.0006	0.0005			0.0006	0.0006				

(c) Based on the worst Spotts' criteria

GA				PSO					
Piston	Cylinder	Cost	Time (s)	Piston	Cylinder	Min	Ave	Max	Time (s)
0.0162	0.0151	66.26	350	0.0161	0.0162	65.82	65.82	65.82	82
0.0037	0.0038			0.0039	0.0038				
0.0012	0.0011			0.0011	0.0012				
0.0006	0.0006			0.0006	0.0006				

(d) Based on the worst mean shift or Greenwood and Chase's unified criteria

Table 1. Optimal tolerances allocation using GA and PSO

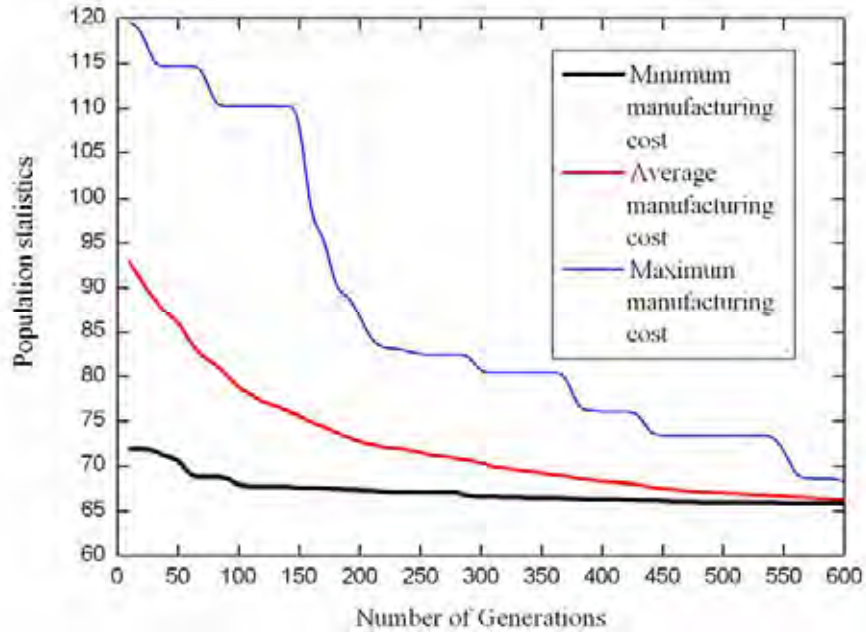


Figure 3. Variation of the minimum, maximum and average of the manufacturing costs with progress of the algorithm (Greenwood and Chase's unified, or estimated mean shift criteria)

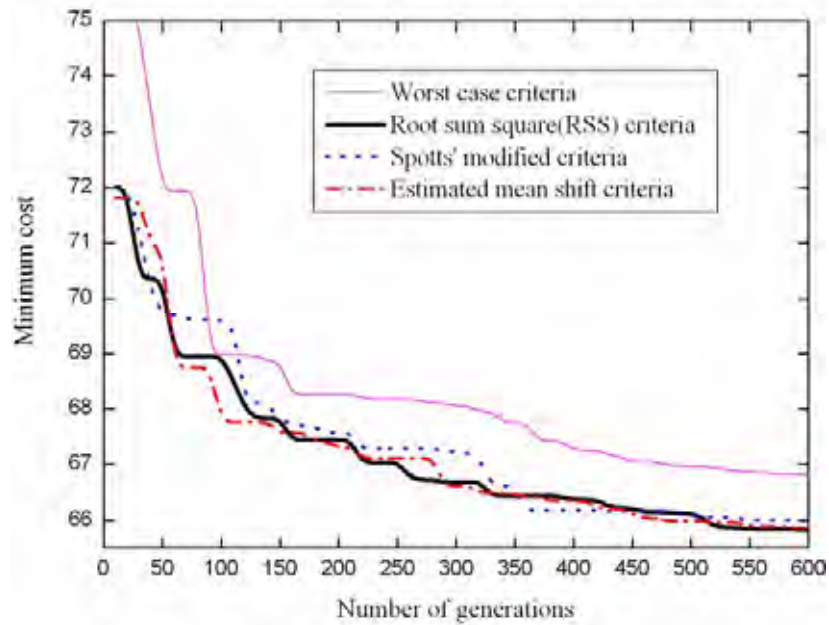


Figure 4. Minimum manufacturing cost in a given number of generations

The proposed PSO algorithm with special constraints handling strategy was used to solve this problem. To validate its efficiency, this new approach was compared with GA in (Singh et al., 2003). In the optimization process of HPSO, we set the population size  $popsiz=80$ , the maximum iteration number  $itermax=600$ . These two parameters are the same as those in GA. The other parameters are set as the common used method. The inertial weight decreases from 0.9 to 0.4 linearly and the accelerated parameters  $c_1=c_2=2$ .

The optimal tolerance allocated using HPSO and GA based on the above four criteria and the corresponding CPU time are listed in Table 1. The computational results clearly indicate that HPSO outperformed GA in the terms of solution quality as well as computational expense. In addition, HPSO is able to find the optimum in each trial, that is, it has significantly larger probability of converging to optimal solutions. It is necessary to point out that one important merit of PSO algorithm is the high precision of the solutions. However, due to the limitation of display capacity of the tables, the entire data are rounded.

The statistical results obtained under the Greenwood and Chase's estimated mean shift criteria are demonstrated in Figure.3. Similar curves can be obtained for other cases. Improvement in the fitness function causes reduction in the assembly manufacturing cost and the amount of infeasibility in subsequent generations. Figure.3 reflects the general behavior about convergence of PSO algorithm. Sharply contrast with GA, the PSO algorithm has consistent convergence. The average and worst fitness are not fluctuant as in GA. Figure.4 demonstrates the minimum manufacturing cost under all four stack-up conditions. The different tendency and position of the curve reveals the difference of the fitness (manufacturing cost).

## 5. Conclusion

Tolerance assignment is very important in product design and machining. The conventional sequentially tolerance allocation suffers from several drawbacks. Therefore, a simultaneous tolerance assignment approach is adopted to overcome these drawbacks. However, the optimization task is usually difficult to tackle due to the nonlinear, multi-variable and high constrained characteristics. In trying to solve such constrained optimization problem, penalty function based methods have been the most popular approach. However, since the penalty function approach is generic and applicable to any type of constraint, their performance is not always satisfactory and consistent. In view of the memory characteristics of PSO, a new constraints handling strategy suit for PSO is designed. This new strategy can adequately utilize the historical information in PSO algorithm. The application on a cylinder-piston assembly example demonstrates its high efficiency and effectiveness.

However, when we attempt to extend the proposed approach to the constrained optimization with large number of complex equality constraints, subtle drawbacks emerged, as the constrained range is so narrow that the equality constraints are hard to satisfy. This problem reveals the new research direction, which is the effective equality constraint handling strategy desirable to develop for PSO based nonlinear programming. Furthermore, powerful local search methods should be introduced to combine with PSO to improve the ability of refined search. In view of its successful application in the above problems especially those engineering ones, PSO can be considered as a general nonlinear constrained optimization tool, and thus could be applied to more engineering optimization problems that can be modeled as nonlinear programming problems.

## 6. References

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